Simplifying Fractions

We used to call it “reducing” fractions, now it is most often called “simplifying” fractions.

To simplify a fraction completely means that the **numerator** (top of the fraction) and **denominator** (bottom of the fraction) share no common factor other than 1. Below are examples of fractions that are and are not simplified.

<table>
<thead>
<tr>
<th>Not Simplified Fractions</th>
<th>Simplified Fractions</th>
</tr>
</thead>
<tbody>
<tr>
<td>6/8</td>
<td>= 3/4</td>
</tr>
<tr>
<td>24/30</td>
<td>= 4/5</td>
</tr>
<tr>
<td>12x³/20x⁵</td>
<td>= 3/5x²</td>
</tr>
</tbody>
</table>

There are several ways to simplify fractions. Frequently students are taught to divide the numerator and the denominator by the same number. You keep doing this until there are no other numbers (other than 1) that you can divide both the numerator and denominator by other than 1.

Examples:

\[
\frac{6}{8} = \frac{6 + 2}{8 + 2} = \frac{3}{4} \\
\frac{24}{30} = \frac{24 + 2}{30 + 2} = \frac{12}{15} \\
\frac{12x^3}{20x^5} = \frac{12x^3 + 4x^3}{20x^5 + 4x^3} = \frac{3}{5x^2} \\
\]

\[
\frac{12 + 3}{15 + 3} = \frac{4}{5} \\
\]
Simplifying Fractions by Decomposition

There are many ways to simplify fractions. One way is to simplify the fractions by breaking down the numerator and the denominator — simplifying by decomposition.

Simplifying fractions by decomposition utilizes what is called the multiplicative identity. The multiplicative identity states that the product of any number and one (1) is the number itself.

\[ a \cdot 1 = a \quad \text{for any number } \quad a \]

By decomposing the numerator and denominator of a fraction you can see the equivalent forms of 1 and then easily simplify the fraction.

Examples:

\[
\frac{6}{8} = \frac{2\cdot3}{2\cdot2\cdot2} = \frac{3}{4}
\]

\[
\frac{24}{30} = \frac{2\cdot2\cdot2\cdot3}{2\cdot3\cdot5} = \frac{2\cdot2}{2\cdot5} = \frac{4}{5}
\]

\[
\frac{12x^3}{20x^5} = \frac{2\cdot2\cdot3\cdotx\cdotx\cdotx}{2\cdot2\cdot5\cdotx\cdotx\cdotx\cdotx\cdotx} = \frac{3}{5x^2}
\]

You can notice based on the examples above that this method also shows you the Greatest Common Factor (GCF) of the numerator and denominator — if you knew this ahead of time you could have divided the numerator and denominator by the GCF to simplify the fraction.

\[
\text{GCF}(6,8) = 2, \quad \text{GCF}(24, 30) = 6, \quad \text{GCF}(12x^3, 20x^5) = 4x^3
\]